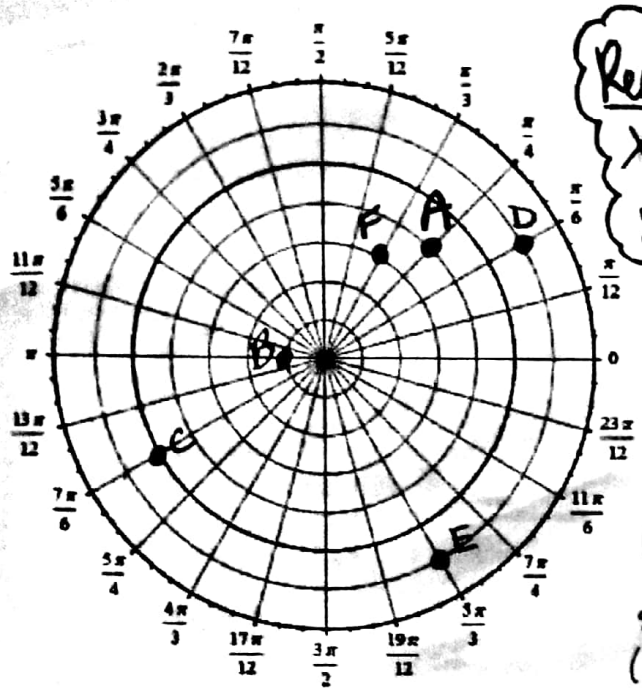
Unit 7 Test Review – Vectors and Polar
Pre-Calculus Honors

Name: Key

1. Plot the following points on the graph below. Then convert each point to a rectangular coordinate.



Remember:
$x = r\cos\theta$
$y = r\sin\theta$

A. $(-3, -135°)$ $= \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

B. $(1, 180°)$ $= (-1, 0)$

C. $\left(-5, -\frac{11\pi}{6}\right)$ $= \left(-\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

D. $\left(-6, \frac{7\pi}{6}\right)$ $= (3\sqrt{3}, 3)$

E. $\left(6, \frac{5\pi}{3}\right)$ $= (3, -3\sqrt{3})$

F. $(-4, -120°)$ $= (2, 2\sqrt{3})$

A. $(-3, -135)$
$(-3\cos -135, -3\sin -135)$

B. $(1, 180)$
$(1\cos 180, 1\sin 180)$

C. $\left(-5, -\frac{11\pi}{6}\right)$
$\left(-5\cos -\frac{11\pi}{6}, -5\sin -\frac{11\pi}{6}\right)$

D. $\left(-6, \frac{7\pi}{6}\right)$
$\left(-6\cos \frac{7\pi}{6}, -6\sin \frac{7\pi}{6}\right)$

E. $(6, 5\pi/3)$
$(6\cos 5\pi/3, 6\sin 5\pi/3)$

F. $(-4, -120)$
$(-4\cos -120, -4\sin -120)$

2. Convert the following from rectangular form to a polar equation, and solve for r.

Remember:
$x = r\cos\theta$; $y = r\sin\theta$; $r^2 = x^2 + y^2$; $\tan\theta = \frac{y}{x}$

a) $2x^2 + 2y^2 = 3$

$2(x^2 + y^2) = 3$

$2r^2 = 3$

$\sqrt{r^2} = \sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$r = \pm\frac{\sqrt{6}}{2}$

b) $2xy = 1$

$2r\cos\theta r\sin\theta = 1$

$r(2\cos\theta\sin\theta) = 1$

$r = \frac{1}{2\cos\theta\sin\theta}$

c) $x = 9$

$r\cos\theta = 9$

$r = \frac{9}{\cos\theta}$

d) $7x - 3y = 8$

$7r\cos\theta - 3r\sin\theta = 8$

$r(7\cos\theta - 3\sin\theta) = 8$

$r = \frac{8}{7\cos\theta - 3\sin\theta}$

3. Convert the following to rectangular equations.

Again... Remember: $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}(y/x)$
$x = r\cos\theta$; $y = r\sin\theta$; $r^2 = x^2 + y^2$; $\tan\theta = \frac{y}{x}$

a) $r\sin\theta = 2$

$y = 2$

b) $r = -3\sec\theta$

$\cos\theta \cdot r = -3\frac{1}{\cos\theta} \cdot \cos\theta$

$r\cos\theta = -3$

$x = -3$

c) $r + 8\sin\theta = 4\cos\theta$ | r

$r^2 + 8r\sin\theta = 4r\cos\theta$

$x^2 + y^2 + 8y = 4x$

Remember... complete the square!

$x^2 - 4x + 4 + y^2 + 8y + 16 = 0 + 4 + 16$

$\left(-\frac{4}{2}\right)^2 = (-2)^2$ $\left(\frac{8}{2}\right)^2 = (4)^2$

$(x - 2)^2 + (y + 4)^2 = 20$

d) $\theta = 4$

$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(4)$

$\frac{y}{x} = \tan 4$

$y = x\tan 4$

4. List 3 different ways to represent the plotted points. Please list 1 point with a +r and 1 point with a −r.

Point A$\left(-1, \frac{5\pi}{6}\right)$

$(+2\pi) = \left(-1, \frac{17\pi}{6}\right)$

$(+\pi$ but change sign$) = \left(1, \frac{11\pi}{6}\right)$

$(-\pi$ but change sign$) = \left(1, -\frac{\pi}{6}\right)$

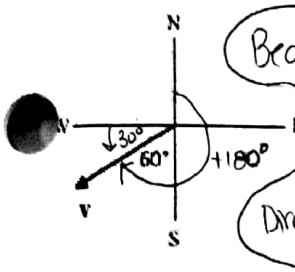
Point B $\left(3, \frac{3\pi}{4}\right)$

$(+2\pi) = \left(3, \frac{11\pi}{4}\right)$

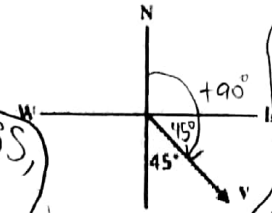
$\rightarrow = \left(-3, \frac{7\pi}{4}\right)$

$\rightarrow = \left(3, -\frac{\pi}{4}\right)$

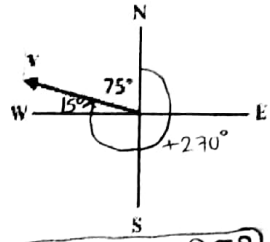
5. State the bearing AND the direction angle for the vectors shown below.



Bearing 240° AND 30° S of W, W 30° S, OR Direction angle: 210° from origin



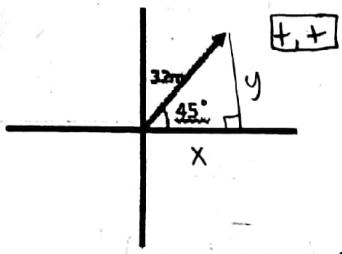
Bearing 135° AND 45° S of E, E 45° S, 45° E of S, S 45° E, OR Direction angle: 315° from origin



Bearing 285° AND 15° N of W, W 15° N, OR Direction Angle: 165° from origin

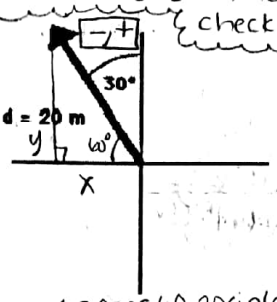
6. Write each of the following vectors in component form.

Remember: $\langle r\cos\theta, r\sin\theta \rangle$ and don't forget to check your signs!



$\langle 32\cos 45, 32\sin 45 \rangle$

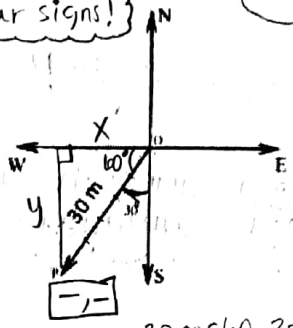
$\langle 16\sqrt{2}, 16\sqrt{2} \rangle$



$\langle 20\cos 60, 20\sin 60 \rangle$

$\langle -10, 10\sqrt{3} \rangle$

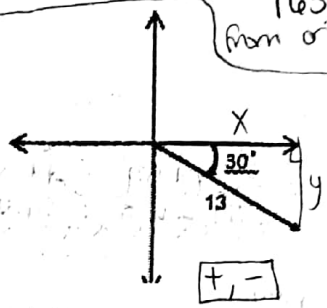
don't forget sign since in Q2!



$\langle 30\cos 60, 30\sin 60 \rangle$

$\langle -15, -15\sqrt{3} \rangle$

don't forget signs since in Q3!



$\langle 13\cos 30, 13\sin 30 \rangle$

$\langle \frac{13\sqrt{3}}{2}, -\frac{13}{2} \rangle$

don't forget sign since in Q4!

7. Consider the vectors **v** = 5**i** – 2**j** and **w** 4**i** – **j**

A) Find $|v| = \sqrt{5^2+(-2)^2} = \sqrt{29}$

Remember: magnitude $= \sqrt{a^2+b^2}$

B) Find 2**w** – 5**v**

Remember: linear combination! component form!

$2(4i-j) - 5(5i-2j) = 8i - 2j - 25i + 10j = -17i + 8j$ OR $\langle -17, 8 \rangle$

C) Find the unit vector in the same direction as **v**.

Remember: to get unit vector, must divide vector by magnitude so magnitude = 1!

$\|v\| = \sqrt{29}$ unit vector $= \frac{\langle 5, -2 \rangle}{\sqrt{29}} = \langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \rangle = \langle \frac{5\sqrt{29}}{29}, -\frac{2\sqrt{29}}{29} \rangle$

then simplify!

8. Given that P(3, -4) and Q(2, -1), find:

Remember: 2nd point MINUS 1st point!

A) The position vector $\overrightarrow{PQ}$.

$\langle 2-3, -1-(-4) \rangle = \langle -1, 3 \rangle$

Remember: $\tan^{-1}\left(\frac{y}{x}\right) = \theta$

B) The direction angle of the vector $\overrightarrow{PQ}$.

Graph $\langle -1, 3 \rangle$:

$\theta = \tan^{-1}\left(\frac{3}{-1}\right) = -71.57° = 18.43°$ W of N

Remember: θ must be a direction angle and less than 45°!

9. Given **v** = ⟨3, –2, 7⟩ and **w** = ⟨0, 5, –2⟩, find:

Remember: Dot Product $v_1 \cdot w_1 + v_2 \cdot w_2$

A) $v \cdot w = 3 \cdot 0 + (-2) \cdot 5 + 7 \cdot (-2)$
$= 0 - 10 - 14$
$= -24$

B) The angle between the two vectors.

Remember: $\cos^{-1}\left(\frac{u \cdot v}{\|u\|\|v\|}\right)$

$v \cdot w = -24$

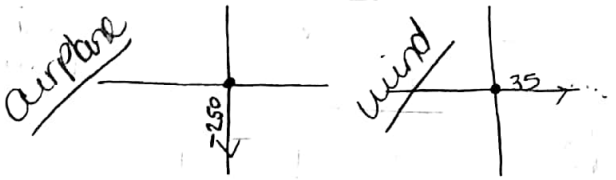
$\|v\| = \sqrt{3^2+(-2)^2+7^2} = \sqrt{62}$

$\|w\| = \sqrt{0^2+5^2+(-2)^2} = \sqrt{29}$

$\cos^{-1}\left(\frac{v \cdot w}{\|v\|\|w\|}\right) = \cos^{-1}\left(\frac{-24}{\sqrt{62}\sqrt{29}}\right) = 124.47°$

10. An airplane travels 250 mph due south. There is a steady 35 mph wind with a bearing of 90°.

A) Write the component form of the velocity vector of the airplane (without wind).



$\langle 0, 250 \rangle$

B) Write the component form of the velocity vector of the airplane. (with wind...)

airplane → $\langle 0, -250 \rangle$

\+ wind → $+ \langle 35, 0 \rangle$

Component form $\langle 35, -250 \rangle$

C) Write the component form of the actual velocity vector of the airplane.

Same answer ...typo!

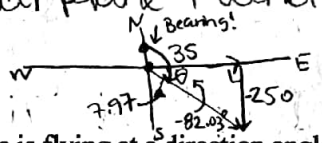
D) What is the actual ground speed of the airplane?

$\|v\| = \sqrt{35^2 + (-250)^2} = 252.44$ mph

E) What is the actual compass bearing of the airplane?

Remember: Connect Tail to Tip

airplane + wind



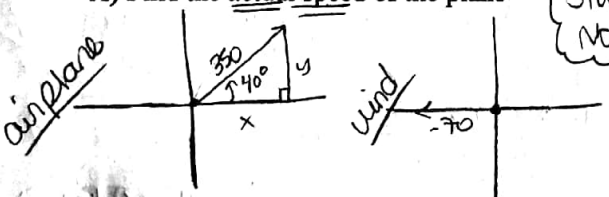
$\theta = \tan^{-1}\left(\frac{-250}{35}\right) = -82.03° =$ Bearing 172.03°

11. An airplane is flying at a direction angle of 40° at 350 mph. A wind is blowing west at 70 mph.

Start at 0° Not NORTH since No directions are given!

* must follow all steps from above (#10)!

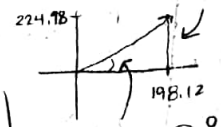
A) Find the actual speed of the plane



$\|v\| = \sqrt{198.12^2 + 224.98^2} = 299.78$ mph

$\langle 350\cos 40, 350\sin 40 \rangle + \langle -70, 0 \rangle = \langle 198.12, 224.98 \rangle$

B) Find the actual bearing of the plane



airplane + wind

$\theta = \tan^{-1}\left(\frac{224.98}{198.12}\right) = 48.63° =$ Bearing 41.37°

C) Find the actual directional angle of the plane

41.37° E of S

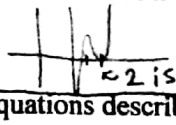
Direction Angle: 48.63° from origin

CUMULATIVE REVIEW SECTION:

Find ALL roots of the polynomial given. Leave ALL answers exact (NO DECIMALS!). Show work!

12. $g(x) = x^4 + x^3 - 19x^2 + 32x - 12$

Graph!



← 2 is EVEN!

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -19 & 32 & -12 \\ & \downarrow & 2 & 6 & -26 & 12 \\ \hline 2 & 1 & 3 & -13 & 6 & \boxed{0} \\ & \downarrow & 2 & 10 & -6 & \\ \hline & 1 & 5 & -3 & \boxed{0} & \end{array}$$

so ÷ by 2 again!

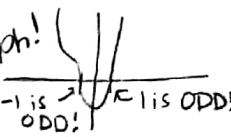
x^2+5x-3 use Quadratic Formula!

$$\frac{-5 \pm \sqrt{5^2-4(1)(-3)}}{2(1)}$$

$$\frac{-5+\sqrt{37}}{2} \qquad \frac{-5-\sqrt{37}}{2}$$

13. $h(x) = x^4 + 4x^3 + 4x^2 - 4x - 5$

Graph!



-1 is ODD! ← 1 is ODD!

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & 4 & -4 & -5 \\ & \downarrow & 5 & 9 & 5 & \\ \hline -1 & 1 & 5 & 9 & 5 & \boxed{0} \\ & \downarrow & -1 & -4 & -5 & \\ \hline & 1 & 4 & 5 & \boxed{0} & \end{array}$$

x^2+4x+5 use Quadratic Formula!

$$\frac{-4 \pm \sqrt{4^2-4(1)(5)}}{2(1)}$$

$$\frac{-4+\sqrt{-4}}{2} \qquad \frac{-4-\sqrt{-4}}{2}$$

$$= \frac{-4+2i}{2} \qquad = \frac{-4-2i}{2}$$

Final Answer: 1, -1, $-2 \pm i$

Final Answer: 2 multiplicity of 2 and $\frac{-5 \pm \sqrt{37}}{2}$

Write the equations described (SHOW WORK):

14. A polynomial in expanded form with roots 4 and -3 +2*i*.

$(x-4)(x-(-3+2i))(x-(-3-2i))$

$(x-4)(x+3-2i)(x+3+2i)$ ← *F.O.I.L!

$(x-4)(x^2+6x+9+4) = x^3+6x^2+13x-4x^2-24x-52$

$= x^3+2x^2-11x-52$

15. Find **all** asymptotes: $T(x) = \frac{x^3-9}{-x^2+x}$

$\frac{x^3-9}{x(-x+1)}$ → T > B → $x \neq 0, x \neq 1$

$$\begin{array}{r} -x \\ -x^2+x \overline{\big) x^3+0x^2+0x-9} \\ -x^3 \quad +x \\ \hline -x-9 \end{array}$$

(remainder)

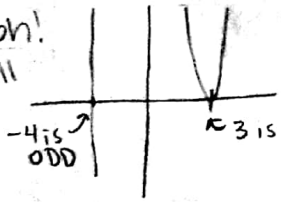
SO HA: none EBA: $y=-x$

VA: $x=0, x=1$

Solve the following inequalities ***by hand*** (show factoring and number line). Write answers in interval notation.

16. $x^3 - 2x^2 - 15x + 36 > 0$

Graph! Still



-4 is ODD

← 3 is EVEN

$$\begin{array}{r|rrrr} -4 & 1 & -2 & -15 & 36 \\ & \downarrow & -4 & 24 & -36 \\ \hline 3 & 1 & -6 & 9 & \boxed{0} \\ & \downarrow & 3 & -9 & \\ \hline 3 & 1 & -3 & \boxed{0} & \\ & \downarrow & 3 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

-5 -1 5

-4 3

− + +

$(-4, 3) \cup (3, \infty)$

* must be "+" to be >0! CAN NOT BE EQUAL HOWEVER!

← Remember: Plug in a value between your zeros!

Solve the following Rational Equations (SHOW WORK).

17. $\frac{1}{x} = \frac{6}{5x} + \frac{1}{1} \cdot \frac{5x}{5x}$

$\frac{1}{x} = \frac{6+5x}{5x}$

ways to Solve

simplify then cross multiply!

Can NOT ÷ by 0!

$5x = 6x + 5x^2$

$0 = 5x^2 + x$

$0 = x(5x+1)$

$\cancel{x=0}$ $x = -\frac{1}{5}$

Extraneous!

18. $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$

simplify then multiply by common denominator!

$\frac{1}{(r-2)(r-5)} \cdot (r-2)(r-5) = \frac{5}{r-2} \cdot (r-2)(r-5)$

$1 = 5r - 25$

$\frac{26}{5} = \frac{5r}{5}$

$\frac{26}{5} = r$

19. Given the function $g(x) = x^4 - 6x^3 + 17x^2 - 24x + 52$ and the fact that $3 - 2i$ is one of the zeros, find all of the other zeros.

← the conjugate is also a zero!

$$\begin{array}{r|rrrrr} 3-2i & 1 & -6 & 17 & -24 & 52 \\ & \downarrow & 3-2i & -13 & 12-8i & -52 \\ \hline 3+2i & 1 & -3-2i & 4 & -12-8i & \boxed{0} \\ & \downarrow & 3+2i & 0 & 12+8i & \\ \hline & 1 & 0 & 4 & \boxed{0} & \\ & x^2 & + & 4 & & \end{array}$$

$(3-2i)(-3-2i)$

$-9 - \cancel{6i} + \cancel{6i} + 4i^2$

$-9+4 \cdot -1$

$-9-4$

-13

$(3-2i)(-12-8i)$

$-36 - \cancel{24i} + \cancel{24i} + 16i^2$

$-36+16 \cdot -1$

$-36-16$

-52

$x^2+4=0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

So all other zeros are: $3+2i, 2i, -2i$