Honors Pre-Calculus Unit 3 Review Sheet   Name:______________________

Formulas that will be given on the test:

$a_n = a_1 + d(n-1)$; $a_n = a_1(r)^{n-1}$; $S_n = \frac{n}{2}(a_1 + a_n)$; $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$; $S = \frac{a_1}{1-r}$

Matrices:

Given the following matrices perform the indicated operations if possible:

$A = \begin{bmatrix} -5 & -2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 3 & -6 \\ 1 & -7 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 2 & 0 \\ -1 & 7 & 1 \end{bmatrix}$ $D = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$ $E = \begin{bmatrix} -4 & 6 \\ 0 & 8 \end{bmatrix}$ $F = \begin{bmatrix} 8 & -6 \\ 1 & 0 \\ 5 & -5 \end{bmatrix}$

1. $A + E$

$\begin{bmatrix} -9 & 4 \\ 3 & 12 \end{bmatrix}$

2. $-3C + 2B$

$\begin{bmatrix} -12 & 0 & -12 \\ 5 & -35 & 1 \end{bmatrix}$

3. $B - 6C$

$\begin{bmatrix} -24 & -9 & -6 \\ 7 & -49 & -4 \end{bmatrix}$

4. $2A + 3C$

DNE

5. AE

$\begin{bmatrix} 20 & -46 \\ -12 & 50 \end{bmatrix}$

6. ED

$\begin{bmatrix} 6 \\ -24 \end{bmatrix}$

7. $CF + A$

$\begin{bmatrix} 29 & -26 \\ 7 & 5 \end{bmatrix}$

8. B^2

DNE

9. AB

$\begin{bmatrix} -2 & -1 & 26 \\ 4 & -19 & -10 \end{bmatrix}$

10. E^2

$\begin{bmatrix} 16 & 24 \\ 0 & 64 \end{bmatrix}$

11. CD

DNE

12. DC

DNE

If $A = \begin{pmatrix} 2p & 3 \\ -4p & p \end{pmatrix}$ and det $A = 14$, find the possible values of p.

$2p^2 + 12p = 14$

$2p^2 + 12p - 14 = 0$

$2(p^2 + 6p - 7) = 0$

$2(p+7)(p-1)$

$\boxed{p = -7 \text{ or } p = 1}$

A and B are 2×2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B.

$\begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix} B = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 22 & 4 \\ -49.5 & -9 \end{bmatrix}$

Let $M = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$, where $a \in \mathbb{Z}$.

(a) Find M^2 in terms of a.

$\begin{bmatrix} a & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} a^2+4 & 2a-2 \\ 2a-2 & 5 \end{bmatrix}$

(b) If M^2 is equal to $\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$, find the value of a.

$2a - \cancel{2} = -4$
$\quad\; \cancel{+2} \quad\; +2$

$2a = -2$

$a = -1$

(c) Using this value of a, find M^{-1} and **hence** solve the system of equations:

$-x + 2y = -3$
$2x - y = 3$

$\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$

$x = 1$

$y = -1$

The talent show committee sold a total of 530 tickets in advance. Student tickets cost \$3 each and the adult tickets cost \$4 each. If the total receipts were \$1740, how many of each type of ticket were sold? Set up a matrix to solve.

X= Student
Y= Adult

$x+y=530$
$3x+4y=1740$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 530 \\ 1740 \end{bmatrix}$$

$x=380$
$y=150$

Rob has 40 coins, all dimes and quarters, worth \$7.60. How many dimes and how many quarters does he have? Set up a matrix to solve.

x=dimes
y= quarters

$x+y=40$
$.10x+.25y=7.60$

$$\begin{bmatrix} 1 & 1 \\ .10 & .25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 40 \\ 7.60 \end{bmatrix}$$

$x=16$
$y=24$

Binomial Expansion:

Use binomial expansion to expand.

$(s-5v)^5$

$1s^5+5s^4(-5v)+10s^3(-5v)^2+10s^2(-5v)^3+5s(-5v)^4+1(-5v)^5$

$s^5-25s^4v+250s^3v^2-1250s^2v^3+3125sv^4-3125v^5$

$(b+2)^7$

$1b^7+7b^6(2)+21b^5(2)^2+35b^4(2)^3+35b^3(2)^4+21b^2(2)^5+7b(2)^6+1(2)^7$

$b^7+14b^6+84b^5+280b^4+560b^3+672b^2+448b+128$

$(d-3b)^3$.

$1d^3+3d^2(-3b)+3d(-3b)^2+1(-3b)^3$

$d^3-9d^2b+27db^2-27b^3$

Find the following terms of the binomial expansion.

$r=4$ $n=8$
5^{th} term of $(4x-3y)^8$

${}_8C_4\,(4x)^{8-4}(-3y)^4$
$=70(256x^4)(81y^4)$
$=\boxed{1451520x^4y^4}$

$r=2$ $n=7$
3^{rd} term of $(b+3)^7$

${}_7C_2\,(b)^{7-2}(3)^2$
$21(b)^5(9)$
$=\boxed{189b^5}$

$r=3$ $n=5$
4^{th} term of $(s-6v)^5$

${}_5C_3\,(s)^{5-3}(-6v)^3$
$10(s)^2(-216v^3)$
$=\boxed{-2160s^2v^3}$

$r=1$ $n=3$
2^{nd} term of $(d-5b)^3$

${}_3C_1\,(d)^{3-1}(-5b)^1$
$3(d)^2(-5b)$
$=\boxed{-15d^2b}$

Sequences:

Describe the pattern in the sequence. Find the next three terms.

13, 15, 17, 19, ... ADDing 2 each time → Arithmetic → 21, 23, 25

4, 8, 16, 32, ... Multiply by 2 each time → Geometric → 64, 128, 256

1, 2, 6, 16, 44,...
ADD the two terms then multiply by 2 → General sequence → 120, 328, 896

625, 250, 100, 40,...
multiply by 2/5 each time → Geometric → 16, 6.4, 2.56

Given the formula $a_n = -4n(n-1)$ Find the first five terms of the sequence.

$a_1=-4(1)(1-1)=0$
$-4(0)$

$a_2=-4(2)(2-1)=-8$
$(-8)(1)$

$a_3=-4(3)(3-1)=-24$
$-12(2)$

$a_4=-4(4)(4-1)$
$-16(3)$
-48

$a_5=-4(5)(5-1)$
$-20(4)$
-80

$\boxed{0,-8,-24,-48,-80}$

Are the following Arithmetic, Geometric, or Neither? Explain your reasoning.

13, 20, 27, 34, ... Arithmetic → $d=+7$

2, −4, −16, −36, ... Neither → No common Differance or Ratio

14, 21, 42, 77, ... Neither → No common Differance or Ratio

$\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \frac{16}{243}, \ldots$ Geometric → $r=2/3$

−2.4, 9.8, 22, 34.2, ... Arithmetic → $d=+12.2$

6, 12, 24, 48, ... Geometric → $r=2$

Write the explicit formula (rule) for the sequence. Then find the fifth term in the sequence.

$a_1=3, r=-3$ $\quad a_n=\{3(-3)^{n-1}\}$ $\quad a_5=243$

$a_1=5, d=-7$ $\quad a_n=\{5-7(d-1)\}$ $\{12-7d\}$ $\quad a_5=-23$

$a_1=120, r=0.3$ $\quad a_n=\{120(.3)^{n-1}\}$ $\quad a_5=.972$

$a_1=1, d=6$ $\quad a_n=\{1+6(n-1)\}$ $\{6n-5\}$ $\quad a_5=25$

Write the recursive formula for the sequences and find the next term. 8, 10, 12, 14, 16, …. $a_{n+1} = a_n + 2$ $a_6 = 18$ 15, 26, 48, 92, 180, …. $a_{n+1} = 2a_n - 4$ $a_6 = 356$	Write the explicit formula (rule) for the sequence and find the indicated term 7, 2, –3, –8, –13, … find a_{14}. $a_n = \{7-5(n-1)\}$ $= \{12-5n\}$ $a_{14} = 12-5(14) = -58$ $\cancel{\frac{1}{2}}\frac{2}{4}, \frac{3}{7}, \cancel{\frac{1}{3}}\frac{4}{12}, \frac{5}{19}, \cancel{\frac{3}{14}}\frac{6}{28}, \ldots$ find a_{14}. $1+3$ $4+3$ $9+3$ $16+3$ $25+3$ $a_n = \left\{\frac{n+1}{n^2+3}\right\}$ $a_{14} = \frac{14+1}{14^2+3} = \frac{15}{199}$

The table shows the predicted growth of a particular bacteria after various numbers of hours. Write an explicit formula (a rule) for the sequence of the number of bacteria.

Hours (n)	**1**	**2**	**3**	**4**	**5**
Number of Bacteria	**19**	**38**	**57**	**76**	**95**

$+19$ $+19$ $+19$ $+19$

$\{a_n\} = \{19+19(n-1)\}$
$= \{\cancel{19} + 19n - \cancel{19}\}$
$= \{19n\}$

Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.

$\frac{15}{\text{0 Bounces}}$ $\frac{12.8}{\text{1st Bounce}}$ $\frac{10.8}{\text{2nd Bounce}}$ $\frac{9.2}{\text{3rd Bounce}}$

Orlando is making a design for a logo. He begins with a square measuring 24 inches on a side. The second square has a side length of 19.2 inches, and the third square has a side length of 15.36 inches. Which square will be the first square with a side length of less than 12 inches?

$\frac{24}{\text{1st Square}}$ $\frac{19.2}{\text{2nd Square}}$ $\frac{15.36}{\text{3rd Square}}$ $\frac{12.288}{\text{4th Square}}$ $\frac{9.8304}{\text{5th Square}}$

Series:

Use summation notation (sigma) to write the following series: 49 + 54 + 59 + … for 14 terms. $\sum_{n=1}^{14} 5n+44$ $49+5(n-1)$ 2 + 4 + 6 + 8 + … for 10 terms $\sum_{n=1}^{10} 2n$ $2+2(n-1)$ 6.6 + 15.4 + 24.2 + … for 5 terms. $\sum_{n=1}^{5} 8.8n-2.2$ $6.6+8.8(n-1)$	Evaluate the following series: $\sum_{n=1}^{4}(n+4) = 26$ $\sum_{n=3}^{8} 5n = 165$ 1 + 4 + 16 + 64 + 256 + 1024. $= 1365$ 6 – 24 + 96 – 384 + … to S_7. $\sum_{n=1}^{7} -4n = -112$ 1000 + 500 + 250 + … to S_5. $\sum_{n=1}^{5} 1000\left(\frac{1}{2}\right)^{n-1} = 1937.5$
Does the infinite geometric series diverge or converge? Explain. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \ldots$ Converge $3 + 9 + 27 + 81 + \ldots$ Diverge	Evaluate the infinite geometric series. Round to the nearest hundredth if necessary. $8 + 4 + 2 + \ldots$ $S_n = \frac{8}{1-\frac{1}{2}} = \frac{8}{\frac{1}{2}} = \boxed{16}$ 1 + 0.1 + 0.01 + … $S_n = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = \boxed{\frac{10}{9}}$

For the series find the first and the last term: $\sum_{n=1}^{5}(n+4)$ 5 and 9 $\sum_{n=4}^{7}-4n$ -16 and -28	Use the finite sequence. Write the related series. Then evaluate the series. 26, 29, 32, 35, 38, 41, 44 $26+29+32+35+38+41+44=245$ 7.6, 6.3, 5, 3.7, 2.4, 1.1, –0.2, –1.5 $7.6+6.3+5+3.7+2.4+1.1+-.2+-1.5=24.2$ The sequence 15, 21, 27, 33, 39, ..., 75 has 11 terms. $a_n=\{15+6(n-1)\}=\{6n+9\}$ $\sum_{n=1}^{11}6n+9=\boxed{495}$ The sequence –5, 0, 5, 10, ..., 65 has 15 terms. $a_n=\{-5+5(n-1)\}=\{5n-10\}$ $\sum_{n=1}^{15}5n-10=\boxed{450}$ The sequence 2, 4, 6, 8, ..., 24 has 12 terms. $a_n=\{2n\}$ $\sum_{n=1}^{12}2n=\boxed{156}$

A large asteroid crashed into a moon of a planet, causing several boulders from the moon to be propelled into space toward the planet. Astronomers were able to measure the speed of one of the projectiles. The distance (in feet) that the projectile traveled each second, starting with the first second, was given by the arithmetic sequence 26, 44, 62, 80, ... Find the total distance that the projectile traveled in seven seconds.

+18 +18 +18

$$\sum_{x=1}^{7}[26+18(x-1)]=560$$

Justine earned $17,000 during the first year of her job at city hall. After each year she received a 4% raise. Find her total earnings during the first five years on the job.

17,000 , 23,800 , 33320 , 46648 , 65307.2 = $186075.20

A rubber ball dropped on a hard surface takes a sequence of bounces, each one $\frac{3}{5}$ as high as the preceding one. If this ball is dropped from a height of 10 feet, what is the total vertical distance it has traveled after it hits the surface the 5th time?

$\frac{10}{0\text{time}}$ $\frac{6}{1^{st}\text{ Time}}$ $\frac{18/5}{2^{nd}}$ $\frac{54/25}{3^{rd}}$ $\frac{162/125}{4^{th}}$ $\frac{486/625}{5^{th}}$

10

Dante is making a necklace with 18 rows of tiny beads in which the number of beads per row is given by the series 3 + 10 + 17 + 24 + ... $3+7(n-1)=7n-4$

a. Use summation notation to write the series. Explain what the numbers in the summation notation represent in this situation and how you found the expression used in the summation.

$$\sum_{n=1}^{18}7n-4$$

b. Find the total number of beads in the necklace. Explain your method for finding the total number of beads.

1125

Cumulative Review:

$\lim\limits_{x\to-5}\frac{x^2+10x+25}{25-x^2} = \frac{0}{0}$ Factor, Cancel, Plugin!

$= \frac{(x+5)\cancel{(x+5)}}{(5-x)\cancel{(5+x)}} = \lim\limits_{x\to-5}\frac{x+5}{5-x} = \frac{-5+5}{5++5} = \frac{0}{10} = \boxed{0}$

$\lim\limits_{x\to2}\frac{x^3-2x^2+x-2}{x-2} = \frac{0}{0}$ Factor, Cancel, Plugin!

$\underline{2}|\ 1\ \ -2\ \ 1\ \ -2$

$\downarrow\ \ \ 2\ \ \ 0\ \ \ 2$

$1x^2\ \ 0x\ \ 1\ \ \boxed{0}$

$\frac{\cancel{(x-2)}(x^2+1)}{\cancel{x-2}} = \lim\limits_{x\to2} x^2+1 = 2^2+1 = \boxed{5}$

$\lim\limits_{h\to0}\frac{4(x+h)-7-(4x-7)}{h}$

$\frac{\cancel{4x}+h\cancel{-7}\cancel{-4x}\cancel{+7}}{h} = \frac{h}{h} = 1$

$\lim\limits_{x\to4}\frac{3x^2-5x-12}{x-3}$

-36 sub -5

$1\cdot36$
$2\cdot18$
$3\cdot12$
$4\cdot9$

$4-9=-5$

$(x+\frac{4}{3})(x-\frac{9}{3})$

$\frac{(3x+4)\cancel{(x-3)}}{\cancel{x-3}} = \lim\limits_{x\to4} 3x+4 = 3(4)+4$

$12+4$

16

An airplane travels 250 mph due south. There is a steady 35 mph wind with a bearing of 90°.

A) Write the component form of the velocity vector of the airplane (without wind).

250

$\langle 0, -250\rangle$

B) Write the component form of ~~of the velocity vector of the airplane~~. wind

90°

35

$\langle 35, 0\rangle$

C) Write the component form of the actual velocity vector of the airplane.

$\langle 35, -250\rangle$

D) What is the actual ground speed of the airplane?

$\sqrt{35^2+250^2} = \boxed{252.44\text{mph}}$

E) What is the actual compass bearing of the airplane?

82.03

$\tan^{-1}\left(\frac{-250}{35}\right)$ Bearing $= 90+82.03$ $\boxed{=172.03}$

An airplane is flying at a direction angle of 40˚at 350 mph. A wind is blowing west at 70 mph.

A) Find the actual speed of the plane

$\langle 198.12, 224.98\rangle$

350

40

$\langle 350\cos40, 350\sin40\rangle$

70

$\langle -70, 0\rangle$

$\sqrt{198.12^2+224.98^2}$

Speed $= 299.77$

B) Find the actual bearing of the plane

48.63

$\tan^{-1}\left(\frac{224.98}{198.12}\right)$

$\boxed{\text{Bearing} = 41.37°}$

C) Find the actual directional angle of the plane

$\boxed{48.63°}$

Identify the type of conic .Change each to standard form and graph

For a **parabola,** find the a) Vertex, b) Focus, c) Directrix, d) Focal Width

For an **ellipse**, find the a) Center, b) Vertices, c) Foci, d) Major Axis
For a **hyperbola**, find a) Center, b) Vertices, c) Foci, d) Asymptotes

7. $4x^2 + 9y^2 - 16x - 36y + 16 = 0$

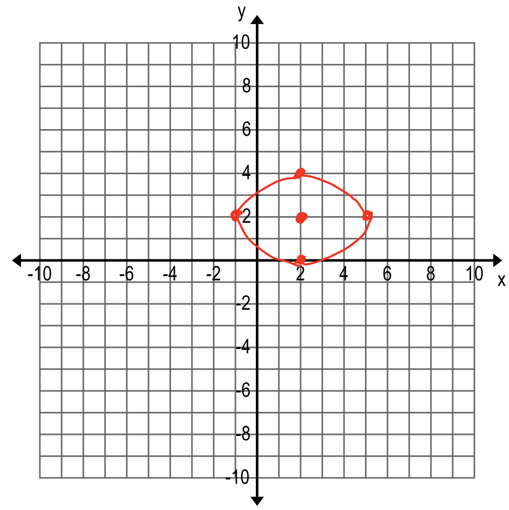
$4(x^2-4x+\underline{4}) + 9(y^2-4y+\underline{4}) = \cancel{-16}$ $+16$ $+36$

$\frac{4(x-2)^2}{36} + \frac{9(y-2)^2}{36} = 1$

$\frac{(x-2)^2}{9} + \frac{(y-2)^2}{4} = 1$

$a=3$ $\quad c^2 = 9-4 = 5$
$b=2$
$c=\sqrt{5}$

Name Ellipse
a) $(2,2)$
b) $(5,2)\ (-1,2)$
c) $(2\pm\sqrt{5},2)$
d)



8. $4x^2 + 24x - 9y^2 - 36y - 36 = 0$

$4(x^2+6x) - 9(y^2+4y) = 36$

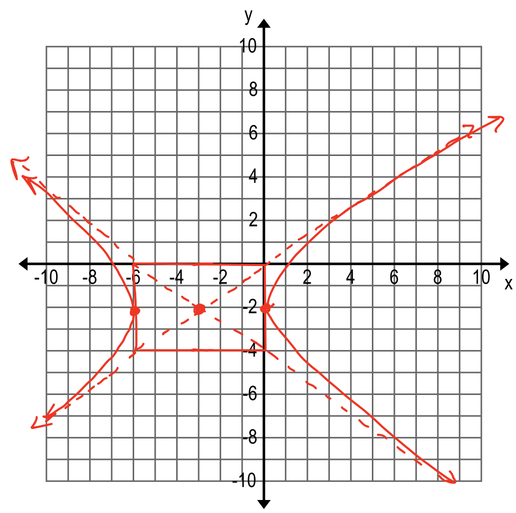
$4(x^2+6x+9) - 9(y^2+4y+4) = \cancel{36}\ 36$ $+36$

$\frac{4(x+3)^2}{36} - \frac{9(y+2)^2}{36} = 1$

$\frac{(x+3)^2}{9} - \frac{(y+2)^2}{4} = 1$

Name hyperbola
a) $(-3,-2)$
b) $(0,-2)\ (-6,-2)$
c) $(-3\pm\sqrt{13},-2)$
d) $y+2 = \pm\frac{2}{3}(x-3)$

$a=3$ $\quad c^2 = 9+4$
$b=2$ $\quad c^2 = 13$
$c=\sqrt{13}$



9. $y^2 - 8x - 6y - 7 = 0$

$y^2 - 6y + 9 - 9 - 7 = 8x$

$(y-3)^2 - 16 = 8x$

$x = \frac{1}{8}(y-3)^2 - 2$

Name Parabola
a) $(-2,3)$
b) $(0,3)$
c) $x=-4$
d)

$\frac{1}{8} = \frac{1}{4p}$

$4p = 8$

$p = 2$