**IB Math SL Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Review – Topic 6, Part 1 Differential Calculus**

*f(x) = position*

*f’(x) = velocity*

*f”(x) = acceleration*

*Differentiating a polynomial: If f(x) = axn + bxm, then f’(x) = anxn-1 + bmxm-1*

*Chain Rule: Use blobs 🡪 derivative of the mother function \* derivative of the stuff inside BLOB*

*Product Rule: f’(x) = u(x)v’(x) + v(x)u’(x)*

*Quotient Rule: *

*Special Derivatives: The derivative of:*

*ex =*

*ln x =*

*sin x =*

*cos x =*

*tan x =*

**Part I – Non-GDC (Paper 1)**

**1.** Differentiate with respect to *x*:

(a) (*x*2 + l)2.

(b) 1n(3*x* – 1).

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**2.** Part of the graph of the periodic function *f*  is shown below. The domain of *f*  is 0 ≤ *x* ≤15 and the period is 3.



(a) Find

(i) *f* (2);

(ii) *f* ‘(6.5);

(iii) *f* ‘(14).

(b) How many solutions are there to the equation *f* (*x*) = 1 over the given domain?

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) (i) ……………………………………… (ii) ……………………………………… (iii) ………………………………………(b) …………………………………………… |

(Total 6 marks)

**3.** **Figure 1** shows the graphs of the functions *f*1, *f*2, *f*3, *f*4.

 **Figure 2 includes** the graphs of the derivatives of the functions shown in **Figure 1**, *eg* the derivative of *f*1 is shown in diagram (d).

 **Figure 1 Figure 2**





 Complete the table below by matching each function with its derivative.

|  |  |
| --- | --- |
| Function | Derivative diagram |
| *f* 1 | (d) |
| *f* 2 |  |
| *f* 3 |  |
| *f* 4 |  |

(Total 6 marks)

**4.** The velocity, *v* m s−1, of a moving object at time *t* seconds is given by *v* = 4*t*3 − 2*t*.

When *t* = 2, the displacement, *s*, of the object is 8 metres.

Find an expression for *s* in terms of *t*.

…………………………………………………………………………………………………………………………………

…………………………………………………………………………………………………………………………………

…………………………………………………………………………………………………………………………………

(Total 6 marks)

**5.** Let *f* (*x*) = *x*3 − 3*x*2 − 24*x* +1.

 The tangents to the curve of *f* at the points P and Q are parallel to the *x*-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let *N*1 and *N*2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects *N*2;

(ii) the tangent at Q intersects *N*1.

..................................................................................................................................................................................................

..................................................................................................................................................................................................

..................................................................................................................................................................................................

..................................................................................................................................................................................................

..................................................................................................................................................................................................

…………………………………………………………………………………………………………………………………………...

 (Total 6 marks)

**6.** Let *f* (*x*) = 3 cos 2*x* + sin2 *x*.

(a) Show that *f* ′ (*x*) = −5 sin 2*x*.

(b) In the interval  ≤ *x* ≤ , one normal to the graph of *f* has equation *x* = *k*.

Find the value of *k*.

(Total 6 marks)

**Part II – GDC – Paper 2**

**7.** A ball is thrown vertically upwards into the air. The height, *h* metres, of the ball above the ground after *t* seconds is given by

*h* = 2 + 20*t* – 5*t*2, *t* ≥ 0

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that *t* must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find .

(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)

(Total 15 marks)

**8.** Consider the function *h* (*x*) = *x.*

(i) Find the equation of the tangent to the graph of *h* at the point where *x* = *a*, (*a* ≠ 0). Write the equation in the form *y* = *mx* + *c*.

(ii) Show that this tangent intersects the *x*-axis at the point (–4*a*, 0).

(Total 5 marks)

**9.** Let *f* (*x*) = .

(a) Write down the **equation** of the vertical asymptote of *y* = *f* (*x*).

(1)

(b) Find *f* ′(*x*). Give your answer in the form  where *a* and *b* ⋲ .

(4)

(Total 5 marks)



**10.** Consider the function 

 A sketch of part of the graph of *h* is given below.

 The line (AB) is a vertical asymptote.

 The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) Find *h*′(*x*), writing your answer in the form



 where *a* and *n* are constants to be determined.

(4)

(c) Given that , calculate the coordinates of P.

(3)

(Total 8 marks)

**11.** Given the function *f* (*x*) = *x*2 – 3*bx* + (*c* + 2), determine the values of *b* and *c* such that *f* (1) = 0 and *f* (3) = 0.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*.................................................................... |

(Total 4 marks)

**12.** Consider *f* (*x*) = *x*3 + 2*x*2 – 5*x*. Part of the graph of *f* is shown below. There is a maximum point at M, and a point of inflexion at N.

1. Find *f* ′ (*x*).

(3)

(b) Find the *x*-coordinate of M.

(4)

(c) Find the *x*-coordinate of N.

(3)

(d) The line *L* is the tangent to the curve of *f* at (3, 12). Find the equation of *L* in the form *y* = *ax* + *b*.

(4)

(Total 14 marks)

**13.** The diagram shows part of the graph of the curve with equation *y =* e2*x* cos *x.*



 (a) Show that  = e2*x* (2 cos *x* –sin *x*)*.*

(2)

(b) Find .

(4)

 There is an inflexion point at P (*a*, *b*)*.*

(c) Use the results from parts (a) and (b) to prove that:

(i) tan *a = *

(3)

(ii) the gradient of the curve at Pis e2*a*.

(5)

(Total 14 marks)