1. $f'(x) = 1 - x^2$

$f(x) = \int (1 - x^2)\mathrm{d}x = x - \dfrac{x^3}{3} + C$ (A1)

$f(3) = 0 \Rightarrow 3 - 9 + C = 0$ (M1)

$\Rightarrow c = 6$ (A1)

$f(x) = x - \dfrac{x^3}{3} + 6$ (A1)

[4]

2. (a) $f'(x) = 5(3x+4)^4 \times 3 \quad = 15(3x+4)^4$ (A1)(A1)(A1) (C3)

(b) $\int (3x+4)^5 \mathrm{d}x = \dfrac{1}{3} \times \dfrac{1}{6}(3x+4)^6 + c \left(= \dfrac{(3x+4)^6}{18} + c \right)$ (A1)(A1)(A1) (C3)

[6]

3. $s = \int v \, \mathrm{d}t$ (M1)

$s = \dfrac{1}{2}\mathrm{e}^{2t-1} + c$ A1A1

Substituting $t = 0.5$

$\dfrac{1}{2} + c = 10$

$c = 9.5$ (A1)

Substituting $t = 1$ M1

$s = \dfrac{1}{2}\mathrm{e} + 9.5 \ (= 10.9 \text{ to } 3 \ s.f.)$ A1 N3

[6]

4. (a) $\int \dfrac{1}{2x+3}\mathrm{d}x = \dfrac{1}{2}\,\mathrm{In}\,(2x+3) + C \ \left(\text{accept } \dfrac{1}{2}\ln|(2x+3)| + C\right)$ A1A1 N2

(b) $\int_0^3 \dfrac{1}{2x+3}\mathrm{d}x = \left[\dfrac{1}{2}\ln(2x+3)\right]_0^3$

evidence of substitution of limits (M1)

eg $\dfrac{1}{2}\ln 9 - \dfrac{1}{2}\ln 3$

evidence of correctly using $\ln a - \ln b = \ln \dfrac{a}{b}$ (seen anywhere) (A1)

eg $\dfrac{1}{2}\ln 3$

evidence of correctly using $a \ln b = \ln b^a$ (seen anywhere) (A1)

eg $\ln\sqrt{\dfrac{9}{3}}$

$P = 3$ (accept $\ln\sqrt{3}$) A1 N2

[6]

5. evidence of anti-differentiation (M1)

eg $s = \int (6e^{3x} + 4)\,dx$

$s = 2e^{3t} + 4t + C$ A2A1

substituting $t = 0$, (M1)

$7 = 2 + C$ A1

$C = 5$

$s = 2e^{3t} + 4t + 5$ A1 N3

[7]

6. (a) $y = e^{x/2}$ at $x = 0$ $y = e^0 = 1$ $P(0, 1)$ (A1)(A1) 2

(b) $V = \pi \int_0^{\ln 2} (e^{x/2})^2\,dx$ (A4) 4

Notes: *Award (A1) for π*
(A1) for each limit
(A1) for $(e^{x/2})^2$.

(c) $V = \int_0^{\ln 2} e^x\,dx$ (A1)

$= \pi[e^x]_0^{\ln 2}$ (A1)

$= \pi[e^{\ln 2} - e^0]$ (A1)

$= \pi[2 - 1] = \pi$ (A1)(A1)

$= \pi$ (AG) 5

[11]

7. (a) evidence of factorizing 3/division by 3 A1

eg $\int_1^5 3f(x)\,dx = 3\int_1^5 f(x)\,dx,\ \frac{12}{3},\ \int_1^5 \frac{3f(x)\,dx}{3}$

(do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign A1

eg $\int_5^1 f(x)\,dx = -\int_1^5 f(x)\,dx$

$\int_5^1 f(x)\,dx = -4$ AG N0

(b) evidence of correctly combining the integrals (seen anywhere) (A1)

eg $I = \int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx = \int_1^5 (x + f(x))\,dx$

evidence of correctly splitting the integrals (seen anywhere) (A1)

eg $I = \int_1^5 x\,dx + \int_1^5 f(x)\,dx$

$\int x\,dx = \frac{x^2}{2}$ (seen anywhere) A1

$\int_1^5 x\,dx = \left[\frac{x^2}{2}\right]_1^5 = \frac{25}{2} - \frac{1}{2} \left(= \frac{24}{2}, 12\right)$ A1

$I = 16$ A1 N3

[7]

8. $\int_1^a \frac{1}{x}\,dx = 2$ (M1)

$\Rightarrow [\ln x]_1^a = 2$ (M1)

$\Rightarrow \ln a = 2$ (A1)

$\Rightarrow a = e^2$ (A1) (C4)

Note: *If 7.39 given instead of e^2 then deduct [1 mark].*

[4]

9. (a) $\frac{ds}{dt} = 30 - at \Rightarrow s = 30t - a\frac{t^2}{2} + C$ (A1)(A1)(A1)

Note: *Award (A1) for 30t, (A1) for $a\frac{t^2}{2}$, (A1) for C.*

$t = 0 \Rightarrow s = 30(0) - a\frac{(0)^2}{2} + C = 0 + C \Rightarrow C = 0$ (M1)

$\Rightarrow s = 30t - \frac{1}{2}at^2$ (A1) 5

(b) (i) vel $= 30 - 5(0) = 30$ m s^{-1} (A1)

(ii) Train will stop when $0 = 30 - 5t \Rightarrow t = 6$ (M1)

Distance travelled $= 30t - \frac{1}{2}at^2$

$= 30(6) - \frac{1}{2}(5)(6^2)$ (M1)

$= 90$m (A1)

90 < 200 => train stops before station. (R1)(AG) 5

(c) (i) $0 = 30 - at \Rightarrow t = \frac{30}{a}$ (A1)

(ii) $30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200$ (M1)(M1)

Note: *Award (M1) for substituting $\frac{30}{a}$, (Ml) for setting equal to 200.*

$\Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200$ (A1)

$\Rightarrow a = \frac{450}{200} = \frac{9}{4} = 2.25$ m s^{-2} (A1) 5

Note: *Do not penalize lack of units in answers.*

[15]

10. (a) $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ (A1) when $x = \text{e}$, $\frac{dy}{dx} = \frac{1}{\text{e}}$

tangent line: $y = \left(\frac{1}{\text{e}}\right)(x - \text{e}) + 1$ (M1)

$y = \frac{1}{\text{e}}(x) - 1 + 1 = \frac{x}{\text{e}}$ (A1)

$x = 0 \Rightarrow y = \frac{0}{\text{e}} = 0$ (M1)

(0, 0) is on line (AG) 4

(b) $\frac{d}{dx}(x \ln x - x) = (1) \times \ln x + x \times \left(\frac{1}{x}\right) - 1 = \ln x$ (M1)(A1)(AG) 2

Note: *Award (M1) for applying the product rule, and (A1) for* $(1) \times \ln x + x \times \left(\frac{1}{x}\right)$.

(c) Area = area of triangle – area under curve (M1)

$= \left(\frac{1}{2} \times \text{e} \times 1\right) - \int_1^{\text{e}} \ln x\,dx$ (A1)

$= \frac{\text{e}}{2} - [x \ln x - x]_1^{\text{e}}$ (A1)

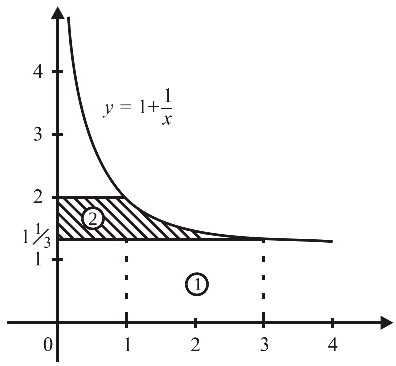
$= \frac{\text{e}}{2} - \{(\text{e} \ln \text{e} - 1 \ln 1) - (\text{e} - 1)\}$ (A1)

$= \frac{\text{e}}{2} - \{\text{e} - 0 - \text{e} + 1\}$

$= \frac{1}{2}\text{e} - 1.$ (AG) 4

[10]

11.



Area $= \int_{1\frac{1}{3}}^{2} x\,dy = \int_{1\frac{1}{3}}^{2} \frac{1}{(y-1)}\,dy$ (M1)(A1)

= [redacted]$n\,(y-1)\Big]_{1\frac{1}{3}}^{2}$

$= \ln 1 - \ln \frac{1}{3}$ (A1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1 + \frac{1}{x}\right) dx = [x + \ln x]_1^3$

$= (3 + \ln 3) – (1 + \ln 1)$ (M1)

$= 2 + \ln 3$ (A1)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 2 + \ln 3 – 2\frac{2}{3} + \frac{2}{3}$ (M1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1 + \frac{1}{x}\right) dx$ (M1)

$A = 3.0986 \ldots$ (G0)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 3.0986 – 2\frac{2}{3} + \frac{2}{3}$ (M1)

$= 1.10$ (3 sf) (A1) (C4)

Notes: *An exact value is required. If candidates have obtained the answer 1.10, and shown their working, award marks as above. However, if they do not show their working, award (G2) for the correct answer of 1.10.*
Award no marks for the giving of 3.10 as the final answer.

[4]

12.

Note: *There are many approaches possible. However, there must be some evidence of their method.*

Area $= \int_0^{k} \sin 2x\,dx$ (must be seen somewhere) (A1)

Using area = 0.85 (must be seen somewhere) (M1)

EITHER

Integrating $\left[\frac{-1}{2}\cos 2x\right]_0^k$

$\left(= \frac{-1}{2}\cos 2k + \frac{1}{2}\cos 0\right)$ (A1)

Simplifying $\frac{-1}{2}\cos 2k + 0.5$ (A1)

Equation $\frac{-1}{2}\cos 2k + 0.5 = 0.85$ ($\cos 2k = – 0.7$)

OR

Evidence of using trial and error on a GDC (M1)(A1)

Eg $\int_0^{\frac{\pi}{2}} \sin 2x\,dx = 0.5$, $\frac{\pi}{2}$ too small etc

OR

Using GDC and solver, starting with $\int_0^{k} \sin 2x\,dx - 0.85 = 0$(M1)(A1)

THEN

$k = 1.17$ (A2) (N3)

[6]