PreCalculus Notes **Geometric Sequences and Series**

Section 9.4 (continued)

**Geometric Sequence:** A pattern where the RATIO of consecutive terms is always the same (something very similar to this was “Geometric Mean” in geometry class).

ex: 2, 6, 18, 54, 162, …

If you divide the second term by the first, the third term by the second, and so on, what do you get? 3

3 is called the COMMON RATIO, denoted r.

In general terms, $r=\frac{a\_{n+1}}{a\_{n}}$

Also, generally speaking, a geometric sequence can be written as: $\{a\_{1}, a\_{1}r, a\_{1}r^{2}, a\_{1}r^{3}, …\}$

**Ex 1: Show that** $\left\{a\_{n}\right\}=\{2^{-n}\}$ **is geometric.**

“Show” means to prove that it is geometric for ANY set of n’s that you plug in… not just a couple of random numbers that you decide to try out. So we need to “show” using the above definition for r and use n and show that we get that r is a constant.

*Solution:* $r=\frac{a\_{n+1}}{a\_{n}} \rightarrow r=\frac{2^{-(n+1)}}{2^{-n}}$ (When dividing equal bases with exponents, subtract the exponents)

 $r=2^{-\left(n+1\right)-(-n)}=2^{-n-1+n}=2^{-1}=\frac{1}{2}$ so r = ½. Since this is a constant, 2-n is geometric.

**Ex 2: Is**$\left\{b\_{n}\right\}=\left\{-3\left(\frac{1}{4}\right)^{n}\right\}$ **a geometric sequence?**

*Solution:* $r=\frac{b\_{n+1}}{b\_{n}} \rightarrow r=\frac{-3\left(\frac{1}{4}\right)^{n+1}}{-3\left(\frac{1}{4}\right)^{n}}=\left(\frac{1}{4}\right)^{n+1-n}=\left(\frac{1}{4}\right)^{1}$ so r = ¼ . Since this is a constant, $-3\left(\frac{1}{4}\right)^{n}$ is geometric.

**Ex 3: Is** $\left\{a\_{n}\right\}=\left\{2n\right\}$ **a geometric sequence?**

*Solution:* $r=\frac{a\_{n+1}}{a\_{n}} \rightarrow r=\frac{2(n+1)}{2n}=\frac{n+1}{n}$ This cannot be simplified further, so r is NOT a constant. Not a geo seq.

**Finding a Rule for a Geometric Sequence:**

Formula: $\left\{a\_{n}\right\}=\left\{a\_{1}r^{n-1}\right\}$

**Ex 4: Find the rule for the sequence listed at the beginning of the notes.**

*Solution:* a1 = 2, r = 3. So, {an} = {2(3)n-1}

**Ex 5: Find the 17th term of the geometric sequence 2, 2/3, 2/9, 2/27, …**

Use the formula with n = 17. Figure out what r is by dividing second term by first: r = 1/3

*Solution*: a17 = 2\*(1/3)17-1 = $\frac{2}{43046721}$.

**Ex 6: Given a1 = 2, r = 3, find the 15th term.**

Plug the given info into the formula and use n = 15.

*Solution:* 9565938

**Adding the first n Terms of a Geometric Sequence:**

Formula: $ S\_{n}=a\_{1}\left(\frac{1-r^{n}}{1-r}\right)$

**Ex 7: Find the general sum of**$ \frac{1}{4}+\frac{2}{4}+\frac{2^{2}}{4}+\frac{2^{3}}{4}+…+\frac{2^{n-1}}{4}$**.**

In order to use the formula, we need r, so divide the second term by the first. r = 2.

*Solution:* $S\_{n}=\frac{1}{4}\left(\frac{1-2^{n}}{1-2}\right)=\frac{1}{4}\left(\frac{1-2^{n}}{-1}\right)=\frac{1-2^{n}}{-4}$

**Ex 8: Find the sum of** $\frac{1}{4}+\frac{2}{4}+\frac{2^{2}}{4}+\frac{2^{3}}{4}+…+\frac{2^{14}}{4}$**.**

Since we already found the general equation for the sum of this sequence, we just need to plug in the value of n that created the last term listed. n =15.

*Solution:* $S\_{15}=\frac{1-2^{15}}{-4}=8191\frac{3}{4}$

OR in calculator: sum(seq(2^(x-1)/4, x, 1, 15))

**Geometric Series:**

A geometric series is the SUM of a geometric sequence that gets infinitely smaller (so 0 < r < 1).

Notation:
$$\sum\_{k=1}^{\infty }a\_{1}r^{n-1}$$

If we use the sum formula listed above, $ S\_{n}=a\_{1}\left(\frac{1-r^{n}}{1-r}\right)$ and consider that 0 < r < 1 and look at what happens as n gets closer to infinity, we get that:

 geometric series = $\frac{a\_{1}}{1-r}$

**Ex 9: Find** $1+\frac{1}{3}+\frac{1}{9}+ …$

We need to use the formula above, so we need r. Divide the second term by the first. r = 1/3.

*Solution:* 1/(1 – 1/3) = 3/2

This should seem like a strange concept – the idea that we are adding and getting a solution to something that never ends… But when happens to the terms toward the “end” of the series? They end up getting so small that they are no longer really affecting the sum when you add them to the previous terms. What is this concept similar to?

**Ex 10: Find 8 + 4 + 2 + …**

Find r by dividing the second term by the first. r = ½

*Solution:* 8/(1 – ½) = 16

**Ex 11:**

$$\sum\_{k=1}^{\infty }2\left(\frac{2}{3}\right)^{k-1}$$

From this notation, we get that a1 = 2 and r = 2/3. Plug them into the formula for a series.

*Solution:* 2/(1 – 2/3) = 6.